# MOTION OF NONINERTIAL SPHERICAL PARTICLES IN A CURVILINEAR FLOW OF A VISCOUS LIQUID AT SMALL REYNOLDS NUMBERS 

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In calculations on aerosol deposition by flow [1] it is usually assumed that the center of a spherical inertia-free particle moves along a streamline near the surface of the body. This assumption has high a priori probability, but it is desirable to test it by experiment, since we do not know the effects of particle rotation (induced by the velocity gradient) on the path. The path shapes are important to the theory of gravitational coalescence [1], of fallout deposition [2], and of the performance of granular and fibrous filters [3].

Consider the motion of a particle (radius r) near a body having a characteristic size $a$ for $x=\mathrm{r} / a \ll 1$ and $(\rho-a) / a \ll 1$ ( $\rho$ is the distance between the centers of particle and collector). This may be considered as motion in a rectilinear flow parallel to the plane of the wall. It has been shown from experiment [4] and theory [5] that, for $\mathrm{R}<1$ (where R is the Reynolds number for the entire flow), the rotation induced by the velocity gradient does not displace the particle from the streamline. However, theory [6] shows that displacement should occur for $\mathrm{R}>10$ (the Segre-Silberberg effect for a cylindrical tube, for example) [7].

If $x \approx 1$ (curvilinear motion around a collector comparable in size with the particle), it is difficult to determine from theory how the rotation will affect the motion and how the particle will affect the flow.

This problem has been examined via a method [8] of determining the flow distribution in a system of parallel cylinders perpendicular to the flow. We photographed the tracks of small particles suspended in glycerol and moving in the brightly lit median plane of a planeparallel cell for cylinders of diameters 7 and 14 mm (Fig. 1). The axes of adjacent cylinders were 30 mm apart. The paths of the particles $\left(x \leq 10^{-2}\right)$ were compared with those of lucite spheres having $r=3 \mathrm{~mm}$, each sphere containing a through cylindrical hole filled with ED-5 epoxide resin. The diameter of the hole was made such that the mean density of a sphere was equal to that of glycerol. The hole also served to indicate the orientation.

The illuminated zone was only 0.5 mm thick, so a sharp image of the surface of a sphere was produced by applying a thin film of aluminum paint. A sphere was injected into the flow with a syringe filled with glycerol via a hole in the side of a cylinder 1 placed on front of cylinder 2 (the one used in the recording). The center of this hole lay exactly in the median plane of the cell, which eliminated rotation of the sphere in a plane parallel to the cylinders and passing through the axis of the lens. The velocity and direction of injection were varied to give paths at different distances from cylinder 2. R for


Fig. 1


Fig. 2
the spheres and cylinders did not exceed 0.05 . The particles and spheres were recorded with stroboscopic illumination, with intervals of 2.7 or 5.4 sec between the $0.2-\mathrm{sec}$ exposures.

The path of the center of a sphere was brought into coincidence with that of a particle at the same flow speed with the flow direction the same for both and the edges of the cylinders coincident. Figure $2 \mathrm{a}, \mathrm{b}$ shows that the sphere speed $\mathrm{V}^{\circ}$ and the particle speed $\mathrm{V}^{*}$ were closely similar for $\chi$ of 0.43 and 0.86 . Kuwabara's [9] theoretical equations describe the speed of a small particle, which coincides with the speed of the liquid, and these have been confirmed [8] by experiment, so the photographs readily gave $\lambda=V^{0} / V^{4}$ as a function of the distance of the center of the sphere from the surface of the cylinder. The results are as follows for various $\rho^{\prime}=(\rho-\alpha) / \mathrm{r}$ at $\theta=90^{\circ}$ :

$$
\begin{aligned}
& x=0.86 \quad \rho^{\prime}=1.151 .421 .521 .95 \quad 2.20 \\
& \lambda=0.98 \quad 0.980 .991 .021 .00 \\
& x=0.43 \quad \rho^{\prime}=1.20 \quad 1.381 .46 \quad 1.60 \\
& \lambda=1.00 \quad 0.981 .99 \quad 0.98
\end{aligned}
$$

However, the motion of the sphere is retarded by the walls of the cell, which are perpendicular to the axes of the cylinders. The theoretical correction [10] to the speed of a sphere in a parabolic flow is $\Delta \mathrm{V} / \mathrm{V}^{*}=(\mathrm{V} / \mathrm{h})^{2} / 3$, in which 2 h is the distance between the walls; in this case it is $2.5 \%$, which is comparable with the $2 \%$ error in locating the center of the sphere in the photographs.

The lines dd' in Fig. 2 show the magnitude and sense of rotation of the sphere; the variation in the sense of rotation with $\theta$ is due to the adjacent cylinders.

It has thus been shown that it is correct to assume that a noninertial particle follows a streamline near the surface of a collector.

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